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Structural Modification to Achieve Antiresonance in Helicopters

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A design method is developed to create an antiresonance (by modifying structural properties) of a vibrating system under sinusoidal loading. A local modification method in which appendant systems are added to the original structure is used to analyze such systems. Since the original system and added systems are treated entirely separately, this method allows for efficient repetitive searching until the appendant system produces a meaningful reduction in vibration. Finally, the direct design of appendant structures to create antiresonance is presented. These methods are illustrated by numerical results obtained for a 44-degree-of-freedom elastic line helicopter model.

Nomenclature

C	= damping matrix = $N\times N$
D_0, D	= parameter modification matrix
E	= Young's modulus
F_i	= Leverrier's matrices
f, f_0	= forcing vector = $N\times 1$
G	= square matrix
g_k, h_k	= polynomials
H, H_{BB_i}	= impedance matrix
I	= area moment of inertia
K	= stiffness matrix = $N\times N$
K_A	= stiffness matrix for attached system
K_e	= 4×4 beam element stiffness matrix
L	= beam length
M	= mass matrix = $N\times N$
M_A	= mass matrix for attached system
M_e	= 4×4 beam element mass matrix
m_0	= beam mass density
N	= number of degrees of freedom
R	= receptance matrix = $N\times N$
s	= parameter equal to $1/\alpha$ or $1/\beta$
u, w	= subsets of the response vector
x, x_0	= response vector = $N\times 1$
Y	= impedance matrix of attached system
$Y_{II}, Y_{IB}, Y_{BI}, Y_{BB}$	= partitions of Y
y_0	= response vector for the modified system = $N\times 1$
Z	= impedance matrix = $N\times N$
α	= percent change in stiffness
β	= design parameter indicating percentage of a base value
Δ	= Leverrier's polynomial
θ_i	= coefficients of Leverrier's polynomial

Introduction

EXCESSIVE levels of vibration in helicopters are a major problem for designers. Fatigue of helicopter components, damage to weapons and equipment onboard the aircraft, crew and passenger discomfort, and navigation problems for the pilot are among the adverse effects of vibration. In this paper, numerical methods for the structural modification of the fuselage and for the analysis and design of appendant structures are applied to the problem of helicopter vibration alleviation. In particular, design techniques are developed to achieve desired antiresonances through structural modification.

In the previous literature on helicopter vibration, the research most relevant to the present work has been concerned with structural modification, vibration isolation devices, and antiresonance theory. In Ref. 1 a simple structural modification is made by inserting a linear spring between two points on the fuselage of a 60-degree-of-freedom helicopter model. The relative effectiveness of changing the spring stiffness is studied using various evaluation criteria. The theoretical basis of this study is given in Ref. 2, which is also the precursor of Ref. 3 where the theory is applied to the model of a pilot's seat structure. A comparative study of the method of Ref. 3 and an optimization method based on the forced response strain energy approach is made in Ref. 4. The authors find the latter more favorable for structures with a large number of degrees of freedom. More general methods for efficient redesign based on reanalysis techniques have been developed.[5,6] An extensive review of these generalized reanalysis approaches applicable to finite-element analysis is provided in Ref. 6.

Vibration isolation devices based on antiresonances, notably the Kaman Aerospace Corporation's dynamic antiresonant vibration isolator (DAVI), have been described in the literature. References 7-9 are among the more recent papers on the DAVI. A solution to the antiresonant eigenvalue problem along with new applications of antiresonance theory to helicopter engineering is presented in Ref. 10.

In this paper, a structural design method which treats a structural property, such as stiffness, as a design parameter is used to create an antiresonance on the helicopter fuselage. The required value of the design parameter is found directly as the root of a polynomial. Next a local modification method is described in which systems are added to the fuselage and the total number of degrees of freedom of the modified system becomes larger than that of the original system. The original system, treated entirely separate from the added systems, is only solved once, and as the properties of the added systems are altered the analysis involves only the added structures. Finally, the direct design of a beam attached to the fuselage to create an antiresonance is presented.

Rotor/Fuselage Model

The numerical examples presented in this paper are for the symmetric elastic line helicopter model developed by Rutkowski.[11] Its three components, the rotor, the

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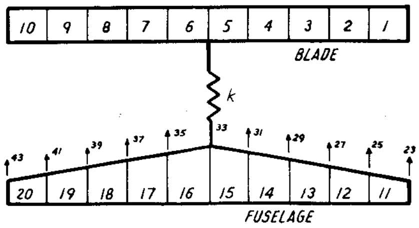

Fig. 1 Elastic line helicopter model.

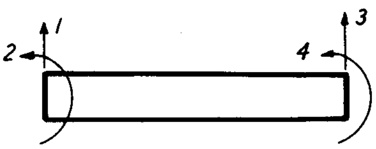

Fig. 2 Beam element geometry.

rotor/fuselage interface, and the fuselage, are shown in Fig. 1. Ten identical beam elements make up the rotor. Each beam element has the 4 degrees of freedom shown in Fig. 2. The rotor stiffness matrix includes the frequency-dependent centrifugal stiffness of the rotating blades. The damping matrix accounts for aerodynamic damping of the blade motion and is also frequency-dependent.

The fuselage is modeled by ten beam elements with linearly varying mass density and stiffness. These beam elements have transverse deflection and rotation, as shown in Fig. 2. Fuselage damping is neglected so that its vibratory behavior is defined by mass and elastic stiffness matrices. The coupling between the rotor and the fuselage is modeled by a stiff linear spring. Numerical values for the structural properties of this helicopter model are given in Table 1.

The beam elements of the model described above are numbered 1-20 in Fig. 1. If j is the element number of a beam element in the rotor blade, then $1 \leq j \leq 10$, and the 4 degrees of freedom (DOF) associated with it are $2j-1$, $2j$, $2j+1$, $2j+2$. The translation DOF of the fuselage are indicated with arrows on Fig. 1. If j is the element number of a beam element of the fuselage, then $11 \leq j \leq 20$, and the DOF associated with it are $2j+1$, $2j+2$, $2j+3$, $2j+4$.

Structural Modification for Antiresonance

The differential equation for the vibration of a mechanical system with N degrees of freedom is

$$M\ddot{x}+C\dot{x}+Kx=f \tag{1}$$

where M, C, and K are the $N \times N$ mass, damping, and stiffness matrices respectively; x is the N vector of generalized displacements; and f is the N vector of generalized forces. For the helicopter model considered in this paper C is a function of rotor frequency. Under sinusoidal loading at frequency ω, the vector x_0 of displacement amplitudes is related to the vector f_0 of force amplitudes by

$$(-\omega^2 M + i\omega C + K)x_0 = f_0 \tag{2}$$

The impedance matrix Z is defined as

$$Z = -\omega^2 M + i\omega C + K \tag{3}$$

and the receptance matrix R is the inverse of Z,

$$R = Z^{-1} \tag{4}$$

The matrices M, C, and K depend on the structural properties of the model chosen to represent the helicopter. Initially we choose a representative set of system parameter values which defines a base receptance matrix R. Then the changes in structural properties are treated as free design parameters and calculated to obtain an antiresonance at a point on the fuselage. In the rest of this section damping is neglected, so that the matrix $C=0$.

For the elastic line helicopter model described in the previous section, let K_e denote a beam element stiffness matrix. Then K_e is a 4×4 matrix which has the form

$$K_e = (EI/L^3)K_0 \tag{5}$$

where E is Young's modulus, I the area moment of inertia, L the length of the beam element, and K_0 a 4×4 symmetric matrix depending only on L. Similarly, the 4×4 beam element mass matrix M_e can be written as

$$M_e = m_0 L M_0 \tag{6}$$

where m_0 is beam density and M_0 is a 4×4 symmetric matrix depending only on L. Now suppose the stiffnesses of some beam elements of the fuselage are changed. The modified form of Eq. (2) is

$$Zy_0 + D_0 y_0 = f_0 \tag{7}$$

where y_0 is the response of the modified system and D_0 is the matrix that is to be added to Z to account for the stiffness change in the beam elements. Since a change in stiffness is effected by changing the quantity EI, and since by Eq. (5) this quantity can be factored out in front of a constant matrix, the matrix D_0 in Eq. (7) can be written as

$$D_0 = \alpha D \tag{8}$$

Table 1 Properties of the beam elements[a]

Element no.	Stiffness 1 $(EI)_1$,[b] lb-ft^2	Stiffness 2 $(EI)_2$,[b] lb-ft^2	Length, ft	Mass density, slug/ft
1-10	20,000	0	5	0.296
11	1,500,000	0	4	1
12	1,500,000	0	4	1
13	1,500,000	3,750,000	4	1
14	5,250,000	3,750,000	4	16
15	9,000,000	3,750,000	4	16
16	12,750,000	−3,750,000	4	16
17	9,000,000	−3,750,000	4	16
18	5,250,000	−3,750,000	4	1
19	1,500,000	0	4	1
20	1,500,000	0	4	1

[a] Interface spring stiffness $= 10^{10}$ (lb/ft). [b] The element elastic stiffness matrix is

$$\frac{(EI)_1}{L^3}\begin{bmatrix} 12 & & & \\ 6L & 4L^2 & \text{Symmetric} & \\ -12 & -6L & 12 & \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} + \frac{(EI)_2}{L^3}\begin{bmatrix} 6 & & & \\ 2L & L^2 & \text{Symmetric} & \\ -6 & -2L & 6 & \\ 4L & L^2 & -4L & 3L^2 \end{bmatrix}$$

$$M = m_0 L \begin{bmatrix} 13/35 & & & \\ 11/210L & 1/105L^2 & & \text{Symmetric} \\ 9/70 & 13/420L & 13/35 & \\ -13/420L & -1/140L^2 & -11/210L & 1/105L^2 \end{bmatrix}$$

where m_0 is the mass density.

where D is to be taken as the base stiffness matrix used for the beam elements in assembling the system stiffness matrix K so that α can be regarded as a percent change in stiffness. Similar considerations apply to beam mass density changes by virtue of Eq. (6).

After obtaining the receptance matrix R of the original structure by inverting the matrix Z, rewrite Eq. (7) as

$$y_0 + \alpha R D y_0 = R f_0 \tag{9}$$

To preserve the symmetry of the helicopter model at least two symmetrically located beam elements, e.g., 14 and 17, must be modified simultaneously. Suppose two nonadjacent beam elements are to be modified. The matrix D is highly sparse; its nonzero entries form an 8×8 matrix at the degrees of freedom where the beam element contributes to the global mass and stiffness matrices. Thus the nonzero entries of RD form an $N \times 8$ matrix. Consequently the product RDy_0 involves only eight coordinates of the full response vector y_0. Let G be the 8×8 matrix obtained from RD by taking the nontrivial entries of the rows $m+j$, $0 \leq j \leq 7$. Define an 8-vector u

$$u_{j+1} = (y_0)_{m+j} \qquad 0 \leq j \leq 7 \tag{10}$$

and an 8-vector w

$$w_{j+1} = (Rf_0)_{m+j} \qquad 0 \leq j \leq 7 \tag{11}$$

Then from Eq. (9) extract a set of eight linear equations in u

$$u + \alpha G u = w \tag{12}$$

so that u is given by

$$u = (I + \alpha G)^{-1} w \tag{13}$$

where I is the 8×8 identity matrix. The matrix inversion in Eq. (13) is conveniently carried out by Leverrier's algorithm,[12] which gives

$$u = s \frac{s^7 F_1 + s^6 F_2 + \cdots + F_8}{s^8 + \theta_1 s^7 + \cdots + \theta_7 s + \theta_8} w \tag{14}$$

where s equals $1/\alpha$, F_i are 8×8 matrices, and θ_i are scalars for $1 \leq i \leq 8$. Since w is a constant vector, Eq. (14) may be rewritten as

$$u_j = s h_j(s) / \Delta(s) \qquad 1 \leq j \leq 8 \tag{15}$$

where $h_j(s)$ is a seventh degree polynomial and

$$\Delta(s) = s^8 + \theta_1 s^7 + \cdots + \theta_7 s + \theta_8 \tag{16}$$

Since the term RDy_0 in Eq. (9) involves only the eight coordinates u_j given by Eq. (15), it is clear that by combining Eqs. (9) and (15), the entire response vector y_0 can be written as ratios of polynomials

$$(y_0)_j = g_j(s) / \Delta(s) \qquad 1 \leq j \leq N \tag{17}$$

Thus if $(y_0)_k$ is to be on antiresonant degree of freedom, then

$$g_k(s) = 0 \tag{18}$$

provided that at least one of the roots s given by Eq. (18) is physically realizable, i.e., such that $\alpha = 1/s > -1$. Equation (17) also provides an efficient way to calculate the response vector y_0 as a function of the design parameter α.

This procedure is easily extended to the case where more than two beam elements are to be modified by the same amount. The polynomial to be solved is then of higher degree since the matrix G in Eq. (13) is of higher order.

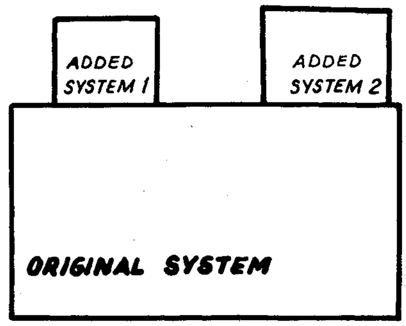

Fig. 3 Modification of the system by two appendant systems.

Local Modification with Increased Degrees of Freedom

Vibrating structures are often modified by appendant dynamic systems. The resulting structure then has additional degrees of freedom and it is natural to seek a method in which the component systems are treated separately to compute the response of the modified system. We will consider the system shown in Fig. 3 where the original structure is modified by adding two new structures to it.

Rewrite Eq. (2) for the original structure as

$$Z x_0 = f_0 \tag{19}$$

After modification, if the original structure is isolated as a free body,

$$Z x = f_0 + f_{B1} + f_{B2} \tag{20}$$

where x is the response vector after modification, the subscript B stands for "boundary," and f_{B1}, f_{B2} are the forces exerted on the original system by the added systems at the points of contact. In the sequel, denote by x_{B_i} the vector of generalized displacements at the interface between the original system and the ith added system. We define the impedance matrix H_{BB_i} associated with the vector x_{B_i} such that

$$f_{B_i} = -H_{BB_i} x_{B_i} \tag{21}$$

Equation (20) becomes

$$Z x = f_0 - H_{BB1} x_{B1} - H_{BB2} x_{B2} \tag{22}$$

where we assume enough zeros have been inserted into the vectors $H_{BB_i} x_{B_i}$ to make them compatible with the dimension of the left-hand side. Next we combine the two vectors x_{B1} and x_{B2} into one vector x_B, and compute a new matrix H such that

$$H x_B = H_{BB1} x_{B1} + H_{BB2} x_{B2} \tag{23}$$

For example, if $x_{B1} = (u_1, u_2, u_3)$ and $x_{B2} = (u_1, u_2, u_4, u_5)$, then $x_B = (u_1, u_2, u_3, u_4, u_5)$; we expand both matrices H_{BB_i} to make them 5×5 by inserting zeros in the appropriate places and obtain H by addition.

Equation (22) can be rewritten as

$$x = Rf - RH x_B \tag{24}$$

since $Z = R^{-1}$. Suppose x_B is an n vector. Since x is an N vector, it is again necessary to assume that N-n zeros have been inserted into the vector RHx_B to make Eq. (24) dimensionally compatible. By a procedure similar to that used to obtain Eq. (12) in the previous section, we extract a set of n linear equations from Eq. (24) to be solved for x_B

$$x_B + G x_B = w \tag{25}$$

where G is an $n \times n$ matrix from the product RH, and w is an n vector. The response vector x is now computed by substituting x_B into Eq. (24).

To complete this description, we indicate how the matrices H_{BB_i}, i.e., the impedance matrices for a free-free structure at the boundary degrees of freedom, can be calculated. Suppose the impedance matrix for the entire structure to be attached is Y. Then

$$Yu = F \tag{26}$$

where u is the vector of generalized displacements and F is the generalized force. We partition the vectors and matrices in Eq. (26) as follows:

$$u \to \begin{Bmatrix} u_I \\ u_B \end{Bmatrix} \tag{27}$$

$$F \to \begin{Bmatrix} F_I \\ F_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_B \end{Bmatrix} \tag{28}$$

$$Y \to \begin{bmatrix} Y_{II} & Y_{IB} \\ Y_{BI} & Y_{BB} \end{bmatrix} \tag{29}$$

where the subscript I designates "interior" and B denotes "boundary." With this notation Eq. (26) becomes

$$Y_{II}u_I + Y_{IB}u_B = 0 \tag{30}$$

$$Y_{BI}u_I + Y_{BB}u_B = F_B \tag{31}$$

Solving Eq. (30) for u_I in terms of u_B and substituting the result into Eq. (31), we obtain

$$(Y_{BB} - Y_{BI}Y_{II}^{-1}Y_{IB})u_B = F_B \tag{32}$$

Equations (21) and (32) give

$$H_{BB} = Y_{BB} - Y_{BI}Y_{II}^{-1}Y_{IB} \tag{33}$$

The vector u_B is identical to the vector x_B of Eq. (25) and since the calculation of x_B is essential to the procedure described above, the internal response u_I of the added structure may immediately be found using Eq. (30):

$$u_I = -Y_{II}^{-1}Y_{IB}u_B \tag{34}$$

Moreover, the characteristic equation of the combined system is

$$\det(I + RH) = 0 \tag{35}$$

This formulation treats the original structure and the added structures separately. The original system need be solved only once. The response vector for the modified system is obtained by solving systems of linear algebraic equations of small order since the number of boundary degrees of freedom is likely to be small compared to the total number of degrees of freedom. Antiresonant vibration absorbers are a special case that can be treated within this framework. The generalization of the method to accept more than two added structures is straightforward. As an application of particular interest in helicopter engineering, the method will be used to study the effect of adding a beam to the fuselage.

Design of Added Structures for Antiresonance

It is possible to combine the ideas presented in the preceding two sections to design an appendant structure such that it will create an antiresonance at a specified location on the fuselage. Let Y be the impedance matrix for the attached system

$$Y = -\omega^2 M_A + K_A \tag{36}$$

where M_A is the mass matrix and K_A is the stiffness matrix for the attached structure. In this section the damping matrix C for the original structure appearing in Eqs. (2) and (3) will be neglected so that $C=0$.

Initially fix M_A and K_A at some base value determined by a set of plausible structural parameter values for the system to be attached. Then take the design value of Y to be a percentage of the corresponding base value of the impedance matrix, that is,

$$Y = \beta(-\omega^2 M_A + K_A) \tag{37}$$

where $\beta > 0$. Now, matrix H_{BB} of Eq. (33) becomes βH_{BB} and Eq. (24) has the form

$$x = Rf - \beta RHx_B \tag{38}$$

while Eq. (25) must be modified to read

$$x_B + \beta G x_B = w \tag{39}$$

But Eq. (39) is of the same form as Eq. (12). Hence by applying to Eq. (39) the procedure described following Eq. (12), x_B can be written as the ratios of polynomials in β. Once x_B is known in terms of β, Eq. (38) yields the remaining coordinates of the response vector in terms of x_B and hence in terms of ratios of polynomials in β. Thus to have antiresonance at coordinate x_k, where

$$x_k = g_k(\beta)/\Delta(\beta) \tag{40}$$

all that remains to be done is to calculate the roots of $g_k(\beta)$. Provided that β is physically realizable (i.e., $\beta > 0$), this procedure will determine the mass and stiffness matrices, βM_A and βK_A, respectively, of the attached structure such that x_k will be an antiresonant coordinate. Note also that Eq. (40) provides an efficient means of calculating the response vector x as a function of the design parameter β.

Numerical Examples

An example of local modification with increased degrees of freedom is shown in Fig. 4. Here a uniform beam is attached rigidly to the fuselage at DOF 29, 30 and 37,38 and is discretized into four elements. Thus 6 degrees of freedom are added to the original helicopter model. Using the method of analysis presented above and varying the stiffness and mass density of the added beam, the designer can quickly determine the structural parameters to satisfy the design objectives.

In all of the numerical examples, the rotor frequency is taken to be 30 rad/s, the excitation frequency 60 rad/s. The sinusoidal force is uniformly distributed over the blades. The

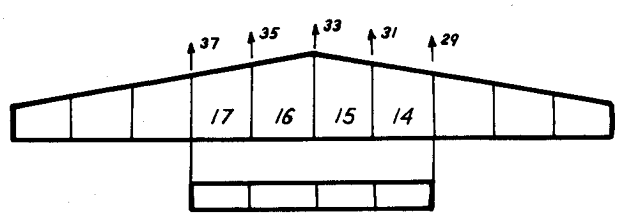

Fig. 4 Helicopter fuselage with added beam.

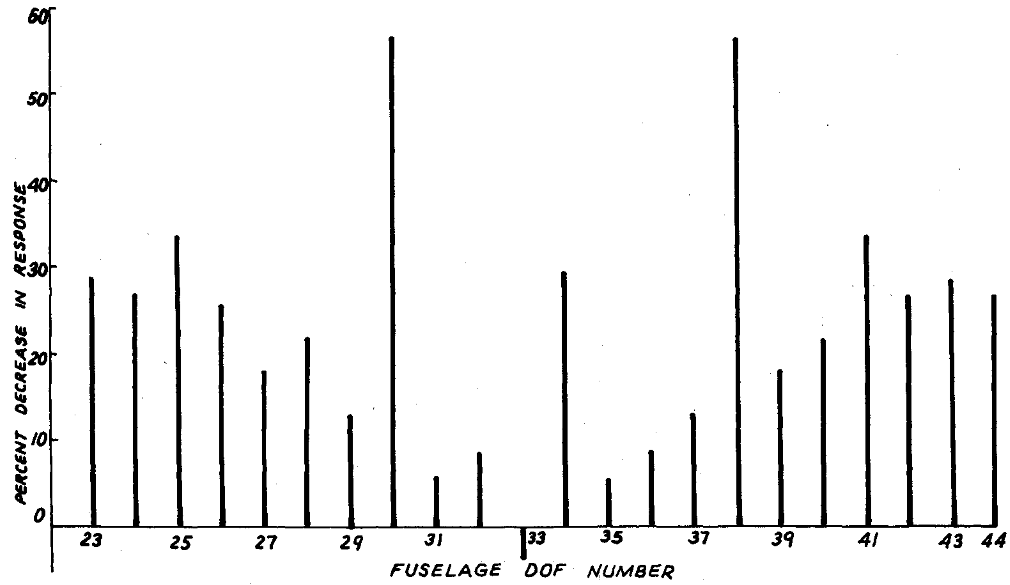

Fig. 5 Percent reduction in vibration achieved by attaching a beam to the fuselage.

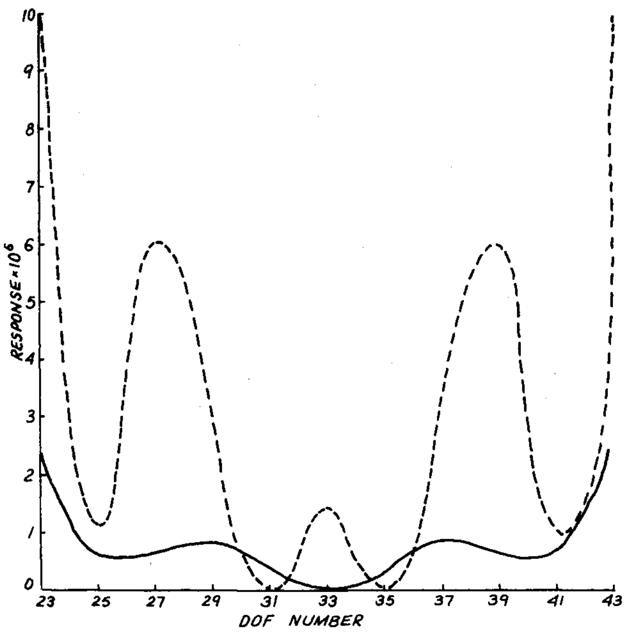

Fig. 6 Fuselage response when elements 12 and 19 are modified to make DOF 31 antiresonant.

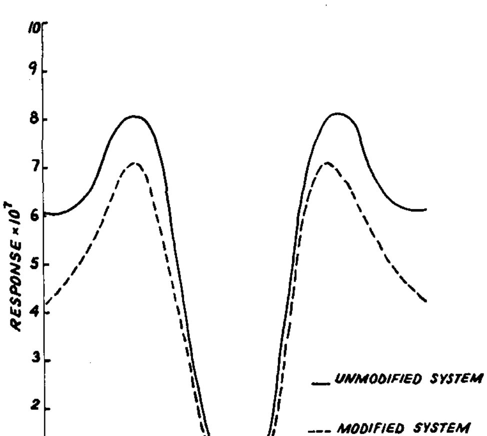

Fig. 7 Fuselage response when a beam is added to the fuselage to make DOF 33 antiresonant.

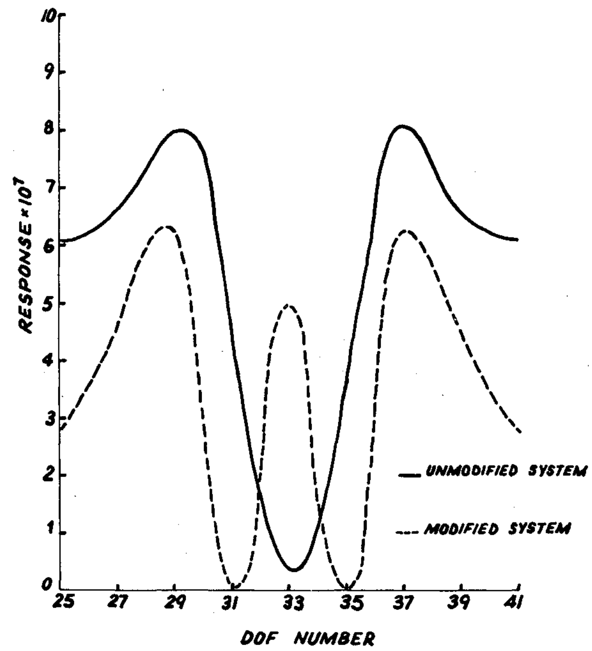

Fig. 8 Fuselage response when a beam is added to the fuselage to make DOF 35 antiresonant.

physical location of the DOF numbers on the helicopter, as well as the beam element numbers, are indicated in Fig. 1, and the numbering scheme is as explained earlier. One particular set of values for beam stiffness and density (Young's modulus $= 3 \times 10^7$ and density $= 0.5$) gives the percent reductions in vibration shown plotted against fuselage DOF in Fig. 5. The percent reduction figures are with respect to fuselage response under the same loading in the absence of the added beam. These reductions vary from 6% to 56% and only one DOF shows an increase (5%).

Figures 6-8 illustrate the design methods for structural modification and for added structures, respectively. These figures show the response of the original system and the modified system vs fuselage DOF numbers. In Fig. 6 the elements 12 and 19 are taken as the elements whose stiffness is to be determined such that DOF 31 becomes antiresonant. The response curve after modification with stiffness reduced by 76% shows that the response at DOF 23,27 becomes amplified and DOF 31 becomes antiresonant as required. Note, however, that this big reduction may weaken the structure to the point of failure. Here an appendant system design, discussed below, would be preferable. It should be noticed that the symmetry of the model automatically makes DOF 35 antiresonant when DOF 31 is made antiresonant by design.

In Fig. 7 the problem is to determine the stiffness and density of the attached beam shown in Fig. 4 such that DOF 33 becomes antiresonant. The response curve after the addition of the beam shows that DOF 33 has become antiresonant while the remaining response levels all lie below the original levels. The attached beam stiffness for this case is 2.628×10^7 and beam density is 0.8760. In Fig. 8 the design requirement is to have DOF 35 antiresonant by attaching the beam shown in Fig. 4 to the fuselage. The response curve after the addition of the beam shows that DOF 35 and DOF 31 have become antiresonant. This is because the modified structure is symmetric with respect to a vertical line drawn through DOF 33. Here the attached beam has stiffness 2.607×10^8 and mass density 8.690.

Conclusions

Two methods of design for antiresonance have been presented. The first method is a structural modification without increasing the number of degrees of freedom of the original structure. The stiffness of a beam element is chosen as a design parameter and the response of the entire structure is determined in closed form as a function of this parameter. The value of this parameter is then found by setting the response function at the desired DOF equal to zero. In the second method, the structural modification increases the number of degrees of freedom of the original structure. The structural properties of the added system are taken as design variables and determined such that they create an antiresonance at a desired DOF. In terms of applicability, the first method is more suitable for designing a new helicopter, whereas the second method is best used to modify an existing helicopter. These methods have been demonstrated for the 44-DOF elastic line helicopter model shown in Fig. 1.

An efficient method of analysis for systems modified by appendant structures has been described and illustrated. The method treats the component systems separately so that the variation of the response as the appendant system parameters change is calculated quickly. For this reason the method is useful in a design by analysis approach in which the analysis is carried out as many times as necessary until a satisfactory set of structural parameter values is found for the appendant system. An example of this as applied to the helicopter model of Fig. 1 has been discussed.

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